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TECHNICAL TRANSLATION

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ON THE COMPUTATION OF THE CIRCULATION OF A GLIDING WING OF LARGE SPAN

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ON THE COMPUTATION OF THE CIRCULATION OF A GLIDING WING OF LARGE SPAN*

By N. M. Monakhov**

In the computation of the circulation of a swept (arrow-shaped) wing of a large span, the "three-quarter chord" method is used. In this method just as in the case of the straight wing, the concept of the lifting line is used; however the boundary condition is satisfied not on the lifting vortex line itself, but at a line displaced from it a distance equal to a half chord.

The method referred to has not been rigorously established. The present paper presents a method of computation of the circulation of a gliding wing of a large span, proceeding from the theory of a lifting surface. In the derivation it is shown that in the determination of circulation with a precision up to magnitudes of the second order of infinitesimal relative to the chord of the wing, the "three-quarter" method is proper.

1. Integral Equation of the Vortex Layer of a Wing.

In the formulation of the integral equation of the vortex layer, the axis ox of a rectangular system of coordinates oxyz is set parallel to the velocity V_∞ of the undisturbed flow; the axis oy is set vertically upward. In the limit, we shall assume the wing infintesimally thin and plane. The angle α between the plane of the wing and the plane $y = 0$, as usual, we

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consider small, and the boundary condition with the surface of the wing we take at the plane $y = 0$.

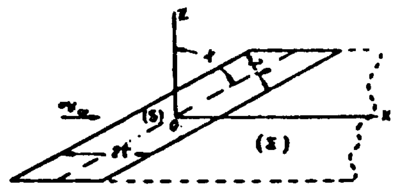

Figure 1.

We designate the intensity of the vortices parallel to the axis oz by $\partial\Gamma/\partial x$, and to the axis ox by $\partial\Gamma/\partial z$. Using the Biot-Savart formula for determining the circulation $\Gamma = \Gamma(x,z)$ we obtain the following integral equation [1]:

$$\iint_{(S+\Sigma)} \left(\frac{\partial\Gamma}{\partial x} \frac{x - x_m}{\rho^3} + \frac{\partial\Gamma}{\partial z} \frac{z - z_m}{\rho^3} \right) dx\, dz = -4\pi V\alpha \cos \chi \qquad (1.1)$$

where

$$\rho = \sqrt{(x - x_m)^2 + (z - z_m)^2}$$

S and Σ are the regions of the projection of the wing and the vortex sheet, respectively, on the plane $y = 0$.

We set the span of the wing in the direction of the axis oz equal to $2l$, and the wing chord, $2t$. In order to simplify the equation (1.1) we introduce the dimensionless slant-angle coordinates ξ, η, and the dimensionless circulation $\overline{\Gamma}$:

$$\frac{x}{l} = \bar{t}\xi + \eta \operatorname{tg} \chi \qquad \frac{z}{l} = \eta \qquad \overline{\Gamma} = \frac{\Gamma}{2\pi l \bar{t} V\alpha \cos^2\chi} \qquad (1.2)$$

In the equations (1.2), $\bar{t} = t/l$, the relative chord of the wing. Taking into account that in the sheet $(\xi > 1)$, $\partial\overline{\Gamma}/\partial\xi = 0$, we transform (1.1), introducing the new variables, to the form:

$$\frac{1}{2} \int_{-1}^{1} \int_{-1}^{1} \frac{\partial\overline{\Gamma}}{\partial\xi} \frac{\overline{t^2}(\xi - \xi_m)^2}{\overline{\rho^3}} \cos \chi \, d\xi \, d\eta$$

$$+ \frac{1}{2} \int_{-1}^{\infty} \int_{-1}^{1} \frac{\partial\overline{\Gamma}}{\partial\eta} \frac{\overline{t^2}(\eta - \eta_m)}{\overline{\rho^3}} \cos \chi \, d\xi \, d\eta = -1 \qquad (1.3)$$

In this equation

$$\bar{\rho} = \sqrt{\frac{(\eta - \eta_m)^2}{\cos^2\chi} + \overline{t^2}(\xi - \xi_m)^2 + 2\bar{t}\ \mathrm{tg}\ \chi(\xi - \xi_m)(\eta - \eta_m)}$$

Carrying out in (1.3) an integration by parts, after transforming, we obtain

$$\frac{\bar{t}\cos\chi}{2}\int_{-1}^{1}\frac{d\bar{\Gamma}}{d\eta}\frac{d\eta}{\eta - \eta_m} - \frac{\cos\chi}{2}\int_{-1}^{1}\int_{-1}^{1}\frac{\partial^2\bar{\Gamma}}{\partial\xi\,\partial\eta}\frac{\bar{\rho}\,d\xi\,d\eta}{(\xi - \xi_m)(\eta - \eta_m)} = -1 \qquad (1.3')$$

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The integral equation obtained for the vortex layer differs from the vortex layer obtained for the straight wing only in the form of the function ρ. Designating by $\bar{b} = \bar{t}\cos\chi$ the half chord of the wing in the direction perpendicular to the axis of the wing, and introducing the new variable

$$\eta = \cos\theta \qquad (1.4)$$

we write (1.3) in the form

$$-\frac{\bar{b}}{2}\int_{0}^{\pi}\frac{d\bar{\Gamma}}{d\theta}\frac{d\theta}{\cos\theta - \cos\theta_m} + \frac{1}{2}\int_{0}^{\pi}\int_{-1}^{1}\frac{\partial^2\bar{\Gamma}}{\partial\xi\,\partial\theta}\frac{\overline{\rho_1}^*d\xi\,d\theta}{(\xi - \xi_m)(\cos\theta - \cos\theta_m)}$$

$$+\frac{1}{2}\int_{0}^{\pi}\int_{-1}^{1}\left(\frac{\partial^2\bar{\Gamma}}{\partial\xi\,\partial\theta}\right)_{\theta=\theta_m}\frac{(\overline{\rho_1} - \overline{\rho_1}^*)d\xi\,d\theta}{(\xi - \xi_m)(\cos\theta - \cos\theta_m)} + \frac{1}{2}\int_{0}^{\pi}\int_{-1}^{1}\left[\frac{\partial^2\bar{\Gamma}}{\partial\xi\,\partial\theta}\right.$$

$$\left.-\left(\frac{\partial^2\bar{\Gamma}}{\partial\xi\,\partial\theta}\right)_{\theta_m}\right]\frac{(\overline{\rho_1} - \overline{\rho_1}^*)d\xi\,d\theta}{(\xi - \xi_m)(\cos\theta - \cos\theta_m)} = -1 \qquad (1.5)$$

where

$$\overline{\rho_1} = \sqrt{(\cos\theta - \cos\theta_m)^2 + b^2(\xi - \xi_m)^2 + 2\bar{b}\sin\chi(\xi - \xi_m)(\cos\theta - \cos\theta_m)}$$

$$\overline{\rho_1}^* = \sqrt{(\cos\theta - \cos\theta_m)^2 + \overline{b^2}(\xi - \xi_m)^2}$$

We expand $\bar{\rho}_1$ in a series:

$$\overline{\rho_1} = \overline{\rho_1}^* + \bar{b}\sin\chi\frac{(\cos\theta - \cos\theta_m)(\xi - \xi_m)}{\overline{\rho_1}^*}$$

$$-\frac{\overline{b^2}\sin^2\chi}{2}\frac{(\cos\theta - \cos\theta_m)^2(\xi - \xi_m)^2}{\overline{\rho_1}^{*3}} + \ldots \qquad (1.6)$$

On substituting $\bar{\rho}_1$ in the form (1.6) into the last integral equation (1.5), the third and following terms of the series after integration will be of the second order of infinitesimal in relation to $\bar{b}$, and can therefore be neglected as compared to it.

With a precision up to the second order of infinitesimal, we shall have:

$$\int_0^{\pi} \int_{-1}^{1} \left[\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta} - \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m}\right] \frac{(\overline{\rho_1} - \overline{\rho_1}^*)\,d\xi\ d\theta}{(\xi - \xi_m)(\cos\theta - \cos\theta_m)}$$

$$= \bar{b}\sin\chi \int_0^{\pi} \int_{-1}^{1} \left[\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta} - \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m}\right] \frac{d\xi\ d\theta}{\overline{\rho_1}^*} \tag{1.7}$$

Taking into account the expansion (1.6) and using (1.4), the third integral in (1.5), with a precision up to the magnitudes of the second order of infinitesimal, can be put in the form:

$$\int_0^{\pi} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m} \frac{(\overline{\rho_1} - \overline{\rho_1}^*)\,d\xi\ d\theta}{(\xi - \xi_m)(\cos\theta - \cos\theta_m)}$$

$$= \bar{b}\sin\chi \int_{-1}^{1} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m} \frac{1}{\overline{\rho_1}^*}\left(\frac{1}{\sqrt{1 - \eta^2}} - \frac{1}{\sqrt{1 - \eta_m^2}}\right) d\xi\ d\eta$$

$$+ \frac{1}{\sqrt{1 - \eta_m^2}} \int_{-1}^{1} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m} \frac{\overline{\rho_1} - \overline{\rho_1}^*}{(\xi - \xi_m)(\eta - \eta_m)}\, d\xi\ d\eta \tag{1.8}$$

As the result of integration, we obtain:

$$\int_{-1}^{1} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m} \frac{d\xi\ d\eta}{\rho_1^* \sqrt{1 - \eta^2}} = 2\int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial\xi\ \partial\theta}\right)_{\theta_m} \sqrt{\frac{\beta}{\eta_m(1 - \beta^2)}}\, K(k)\,d\xi \tag{1.9}$$

where

$$\beta = \frac{1 + \overline{b^2}(\xi - \xi_m)^2 + \eta_m^2}{2\eta_m} - \frac{\sqrt{\left[1 + \overline{b^2}(\xi - \xi_m)^2 + \eta_m^2\right]^2 - 4\eta_m^2}}{2\eta_m}$$

K(k) is a total elliptical integral of the first order, in which the square of the modulus is given by

$$k^2 = \frac{\beta}{\eta_m} \frac{\left[\left(1 - \eta_m\beta\right)^2 + \overline{b}^2\beta^2\left(\xi - \xi_m\right)^2\right]}{\left(1 - \beta^2\right)^2} \tag{1.10}$$

Disregarding sections near the wing tip ($|\eta| = 1$), with a precision to terms of the second order of infinitesimal

$$K(k) = \ln \frac{4}{\sqrt{1 - k^2}} = \ln \frac{4\left(1 - \eta_m^2\right)}{\bar{b}\left|\xi - \xi_m\right| \cdot \eta_m} \tag{1.11}$$

With the assumed degree of precision, the remaining integrals in (1.8) are expressed by elementary functions:

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$$\begin{aligned} \int_{-1}^{1} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta}\right)_{\theta_m} \frac{\bar{\rho}_1 - \bar{\rho}_1^{*}}{(\xi - \xi_m)(\eta - \eta_m)}\, d\xi\, d\eta \\ = \bar{b}\left[2 \sin \chi - \ln \frac{1 + \sin \chi}{1 - \sin \chi} + \sin \chi \ln \frac{4 \sin^2 \theta_m}{\bar{b}^2 \cos^2 \chi}\right] \frac{d\bar{\Gamma}}{d\theta_m} \\ - 2\bar{b} \sin \chi \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta}\right)_{\theta_m} \ln\left|\xi - \xi_m\right| d\xi \end{aligned} \tag{1.12}$$

$$\begin{aligned} \int_{-1}^{1} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta}\right)_{\theta_m} \frac{d\xi\, d\eta}{\bar{\rho}_1^{*}} = \ln \frac{4 \sin^2 \theta_m}{\bar{b}^2} \frac{d\bar{\Gamma}}{d\theta_m} \\ - 2 \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta}\right)_{\theta_m} \ln\left|\xi - \xi_m\right| d\xi \end{aligned} \tag{1.13}$$

Using (1.7) – (1.9) and (1.11) – (1.13), equation (1.5) is written in the following form:

$$\begin{aligned} -1 = \frac{1}{2} \int_{0}^{\pi} \int_{-1}^{1} \frac{\partial^2 \bar{\Gamma}}{\partial \xi\, d\theta} \frac{\bar{\rho}_1^{*} d\xi\, d\theta}{(\xi - \xi_m)(\cos \theta - \cos \theta_m)} - \frac{\bar{b}}{2} \int_{0}^{\pi} \frac{d\bar{\Gamma}}{d\theta} \frac{d\theta}{\cos \theta - \cos \theta_m} \\ + \frac{\bar{b}}{\sin \theta_m}\left(\sin \chi - \ln \sqrt{\frac{1 + \sin \chi}{1 - \sin \chi}} + \sin \chi \ln \frac{4 \sin^2 \theta_m}{\bar{b} \cos \chi}\right) \frac{d\bar{\Gamma}}{d\theta_m} \\ - \frac{\bar{b} \sin \chi}{\sin \theta_m} \int_{-1}^{1} \frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta_m} \ln\left|\xi - \xi_m\right| d\xi + \frac{\bar{b} \sin \chi}{2} \int_{0}^{\pi} \int_{-1}^{1} \left(\frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta} \right. \\ \left. - \frac{\partial^2 \bar{\Gamma}}{\partial \xi\, \partial \theta_m}\right) \frac{d\xi\, d\theta}{\bar{\rho}_1^{*}} \end{aligned} \tag{1.14}$$

Without changing quantities of the first order of infinitesimal, we assume in computing the double integrals in (1.14) $\bar{\rho}_1^{*} = |\eta - \eta_m|$. Carrying out the integration, we obtain:

sponding section of the wing, which we determine later.

For solving the indicated Cauchy problem we construct differential equations of the properties of equation (2.1). Introducing, along with θ_m, the variable $\eta = \cos \theta_m$, we obtain:

$$\int_0^{\pi}\int_{-1}^{1}\frac{\partial^2\overline{\Gamma}}{\partial\xi\,\partial\theta}\,\frac{\overline{\rho_1}^{*}d\xi\,d\theta}{(\xi-\xi_m)(\cos\theta-\cos\theta_m)}=2\int_{-1}^{1}\frac{\partial\overline{\Gamma}}{\partial\xi}\,\frac{d\xi}{\xi-\xi_m} \qquad (1.15)$$

$$\int_0^{\pi}\int_{-1}^{1}\left(\frac{\partial^2\overline{\Gamma}}{\partial\xi\,\partial\theta}-\frac{\partial^2\overline{\Gamma}}{\partial\xi\,\partial\theta_m}\right)\frac{d\xi\,d\theta}{\overline{\rho_1}^{*}}=\int_0^{\pi}\frac{\frac{d\overline{\Gamma}}{d\theta}-\frac{d\overline{\Gamma}}{d\theta_m}}{|\cos\theta-\cos\theta_m|}\,d\theta \qquad (1.16)$$

We note that on the leading edge of the wing ($\xi = -1$), $\partial\overline{\Gamma}/\partial\theta_m = 0$, so that:

In integrating the system (2.3); $\frac{d\overline{\Gamma}}{d\eta}$, Φ_1 as well as $\overline{\Gamma}$, are considered known functions of η.

From the first of the relations (2.3) we obtain the following first integral of the system:

$$\eta + \overline{b}\sin\chi\xi = c_1 \qquad (2.5)$$

The second integral of the system we obtain from consideration of the first and subsequent terms of equations (2.3); in so doing, on the basis of (2.5) with precision up to quantities of the second order of infinitesimal we consider in the subsequent terms of (2.3)

$$\Phi_1(c_1 - \overline{b}\sin\chi\xi) = \Phi_1(c_1) - \frac{\partial\Phi_1}{\partial c_1}\overline{b}\sin\chi\xi$$

$$\frac{d\overline{\Gamma}}{d\eta} = \frac{d\overline{\Gamma}}{dc_1}$$

Considering $\Phi_1(c_1)$, $\frac{\partial\Phi}{\partial c_1}$, $\frac{d\overline{\Gamma}}{dc_1}$ as constant quantities we obtain after integration, with precision up to magnitudes of the second order of infinitesimal

$$\Phi_1(c_1)\left(\sqrt{1-\xi^2}-\text{arc}\cos\xi\right)+\frac{\overline{b}\sin\chi}{\pi}\frac{d\overline{\Gamma}}{dc_1}\left(\pi\xi-\xi\,\text{arc}\cos\xi+\sqrt{1-\xi^2}\right)$$

$$-\frac{1}{\pi}\frac{\partial\Phi_1}{\partial c_1}\overline{b}\sin\chi\left(\frac{\text{arc}\cos\xi}{2}-\sqrt{1-\xi^2}+\frac{1}{2}\xi\sqrt{1-\xi^2}\right)+\overline{\Gamma}(\xi,\eta)=c_2 \qquad (2.6)$$

If we take account of the condition indicated above, $\overline{\Gamma} = \overline{\Gamma}(\eta)$ at $\xi = 1$, we obtain on the basis of (2.6) the following relation between c_1 and c_2'

$$c_2 = \overline{b}\sin\chi\frac{d\overline{\Gamma}}{dc_1} + \overline{\Gamma}(c_1 - \overline{b}\sin\chi) \qquad (2.7)$$

$$\bar{\Gamma}(\xi,\eta) = \bar{\Gamma}(\eta) + \frac{\bar{b}\sin\chi}{\pi}\frac{d\bar{\Gamma}}{d\eta}\left(\xi \operatorname{arc\,cos}\xi - \sqrt{1-\xi^2}\right) - \frac{\Phi_1(\eta)}{\pi}\left(\sqrt{1-\xi^2}\right.$$
$$\left. - \operatorname{arc\,cos}\xi\right) + \frac{1}{\pi}\frac{d\Phi_1}{d\eta}\bar{b}\sin\chi\left[\left(\frac{1}{2}+\xi\right)\operatorname{arc\,cos}\xi\right.$$
$$\left. - \left(1+\frac{\xi}{2}\right)\sqrt{1-\xi^2}\right] \quad (2.8)$$

The problem of determining the vortex layer of the wing can be considered solved if the circulation $\bar{\Gamma}(\eta)$, entering into (2.8) is found.

For determination of $\bar{\Gamma}(\eta)$ the condition is used that $\bar{\Gamma}(\xi,\eta)$ reduces to zero on the leading edge of the wing.

Assuming in equation (2.8) $\xi = -1$, $\bar{\Gamma}(-1,\eta) = 0$, and replacing η with the variable θ, we obtain the following equation for $\bar{\Gamma}(\theta)$:

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$$\bar{\Gamma}(\theta) + \Phi_1(\theta) + \frac{\bar{b}\sin\chi}{\sin\theta}\frac{d\bar{\Gamma}}{d\theta} + \frac{1}{2}\frac{\bar{b}\sin\chi}{\sin\theta}\frac{d\Phi_1}{d\theta} = 0 \quad (2.9)$$

Using (2.2), (2.4), we obtain on the basis of (2.9) the following integral equation for $\bar{\Gamma}(\theta)$:

$$\bar{\Gamma}(\theta_m) = 1 - \frac{\bar{b}}{2}\int_0^{\pi}\frac{d\bar{\Gamma}}{d\theta}\frac{d\theta}{\cos\theta - \cos\theta_m} + \frac{\bar{b}\sin\chi}{2}\int_0^{\pi}\frac{\frac{d\bar{\Gamma}}{d\theta} - \frac{d\bar{\Gamma}}{d\theta_m}}{|\cos\theta - \cos\theta_m|}\,d\theta$$
$$+ \frac{\bar{b}\sin\chi}{\sin\theta_m}\left(\frac{1}{2} - \frac{1}{\sin\chi}\ln\sqrt{\frac{1+\sin\chi}{1-\sin\chi}} + \ln\frac{8\sin^2\theta_m}{\bar{b}\cos\chi}\right)\frac{d\bar{\Gamma}}{d\theta_m} \quad (2.10)$$

The solution $\bar{\Gamma}(\theta)$ of the latter equation can be obtained by any of a number of effective methods [3].

Keeping in mind that with a precision up to magnitudes of the second order of infinitesimal, (2.9) takes the form:

$$\Phi_1(\theta) = -\bar{\Gamma}(\theta) - \frac{\bar{b}\sin\chi}{2\sin\theta}\frac{d\bar{\Gamma}}{d\theta}$$

we obtain the following final formulas for $\bar{\Gamma}(\xi,\theta)$ and the intensity of the vortex layer of a wing along and at right angles to the flow:

$$\Gamma(\xi,\theta) = \frac{\bar{\Gamma}(\theta)}{\pi}\left(\pi + \sqrt{1-\xi^2} - \operatorname{arc\,cos}\xi\right) + \frac{\bar{b}\sin\chi}{2\pi\sin\theta}\frac{d\bar{\Gamma}}{d\theta}(1-\xi)\sqrt{1-\xi^2} \quad (2.8')$$

$$\frac{\partial \overline{\Gamma}}{\partial \xi} = \frac{1}{\pi}\left(\overline{\Gamma}(\theta) + \frac{\overline{b}\,\sin\,\chi}{2\,\sin\,\theta}\frac{d\overline{\Gamma}}{d\theta}\right)\sqrt{\frac{1-\xi}{1+\xi}} - \frac{\overline{b}\,\sin\,\chi}{\pi\,\sin\,\theta}\frac{d\overline{\Gamma}}{d\theta}\sqrt{1-\xi^2} \tag{2.11}$$

$$\frac{\partial \overline{\Gamma}}{\partial \theta} = \frac{1}{\pi}\frac{d\overline{\Gamma}}{d\theta}\left(\pi + \sqrt{1-\xi^2} - \text{arc cos}\ \xi\right) + \frac{\overline{b}\,\sin\,\chi}{2\pi}\frac{d}{d\theta}\left(\frac{d\overline{\Gamma}/d\theta}{\sin\,\theta}\right)(1-\xi)\sqrt{1-\xi^2} \tag{2.12}$$

On the basis of (2.11) we conclude that the intensity of the vortex at a wing section, with precision up to magnitudes of the second order of infinitesimal (except for the wing tip), is distributed according to the law that is valid for plane parallel flow around a parabolic profile.

Since in the determination of circulation around a parabolic profile, the "three-quarter chord" method gives precise results, it may be concluded that its application for computation of swept wings, within the limits of a linear theory, is justified.

The velocity circulation $\overline{\Gamma}(\theta)$ around a wing section, being the solution of equation (2.10), will be found to agree with experiment with a precision up to magnitudes of the second order of infinitesimal, except for the wing tip ($|\eta| = 1$), near which equations (1.18) and (2.10) are not precise.

Evaluation of the basic terms of the equation (1.18) shows that the circulation of a gliding straight wing is determined according to equation (2.10) with practically the same degree of precision, for η changing within the limits:

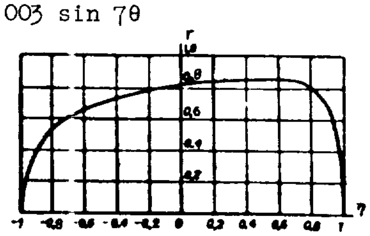
$$-1 + \frac{\overline{b}}{2} < \eta < 1 - \frac{\overline{b}}{2}$$

It follows that in the computation of the circulation of a gliding wing, the distances between sections of the wing should be chosen not less than a quarter chord apart. Ordinarily such distances are not chosen less than that limit. As an example of the application of equation (2.10) the circulation was computed for a wing with relative chord $2\overline{b} = 1/\pi$ for the angle of sweep $\chi = 45^{\circ}$:

$$\overline{\Gamma} = 0.895\ \sin\ \theta + 0.136\ \sin\ 2\theta + 0.120\ \sin\ 3\theta + 0.041\ \sin\ 4\theta$$

$$+\ 0.027\ \sin\ 5\theta + 0.008\ \sin\ 6\theta + 0.003\ \sin\ 7\theta$$

A graph of the function Γ is shown in figure 2.



REFERENCES

[1] N. E. Kochin, I. A. Kibel', N. V. Roze, Teoreticheskaya gidromekhanika [Theoretical hydromechanics], Chap. 1, Gostekhizdat (1948).
[2] N. I. Muskhellishvili, Singularnye integralnye uravneniya [Singular integral equations], Gostekhizdat (1948).
[3] V. V. Golubev, Leklsii poteorii kryla [Lectures on wing theory], Gostekhizdat (1949).

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